NONSTATIONARY TEMPERATURE FIELD IN A POROUS BODY UPON GAS FILTRATION

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A solution is given for the problem of a nonstationary temperature field in spatial porous bodies upon filtration of a thermodynamically ideal gas.

If, in the region of gas filtration, heat sources (sinks) are absent and the thermophysical parameters of the heat-conducting porous material are constant in the range of temperature variation examined (equal to its mean values) then heat transfer in the case of ideal contact is described by the following equation:

$$a^2\Delta T - \gamma \operatorname{div}(\rho \mathbf{v} T) = \frac{\partial T}{\partial t},$$

where $\gamma = c_p/\rho_1 c_1$ has a constant value.

During gas filtration in a nondeforming porous medium the resistance law in the general case can be described after the manner of Khristianovich [1] in the following form:

$$\frac{k}{\mu}\operatorname{grad} P = -\Phi(v)\frac{\mathbf{v}}{v}.$$

In each actual case the form of the function $\Phi(v)$ is determined by the filtration law. One of the forms of the function $\Phi(v)$ most justifiable from the theoretical and experimental points of view is as follow [2]:

$$\Phi(v) = Av + Bv^2,$$

where A and B are empirical coefficients depending on the porous structure of the material and the viscosity of the gas.

For large values of v, which are characteristic for a porous cooling system with strongly heated and coarse bodies, the first term of Eq. (3) is negligibly small compared to the second term. In this case the principal role is played by inertial forces (the resistance is proportional to v^2), and forces of viscosity play a secondary role. Consequently one can assume that the viscosity of the gas is constant (μ = const) and equal to its mean value in the temperature range examined.

Approximating the curve $\Phi(v)$ by the straight line $\bar{\Phi}(v) = a + bv$ in such a way that the divergence between $\bar{\Phi}(v)$ and $\Phi(v)$ in the region of values of v considered is insignificant, we obtain

$$\frac{\Phi(v)}{v} = \operatorname{tg}\zeta = \omega = \text{const},$$

where ζ is the angle of inclination of the straight line $\bar{\Phi}(v)$ to the abscissa v.

We assume that the gas is compressible and thermodynamically ideal, i.e.,

$$\rho = P/RT.$$

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On the strength of the assumptions made, Eq. (2) can be written in the form

$$V = \rho v = -\frac{m}{T} \operatorname{grad} P^2.$$

Here $m = k/2R\mu\omega$ is a constant.

After substituting (4) in Eq. (1) we obtain

$$a^2 \Delta T + \sigma \Delta P^2 = \frac{\partial T}{\partial t},$$

where $\sigma = kc_p/2R\mu\rho_1 c_1\omega$ is a constant.

If the boundary of the porous medium can be split into two parts, corresponding to the entry and exit of gas, at each of which a constant pressure, P_I and P_{II}, respectively, is maintained, then one may assume that $\partial P^2/\partial t \approx 0$ and Eq. (5) is written in the following form:

$$a^2 \Delta \varphi = \frac{\partial \varphi}{\partial t},$$

where $\varphi = T + nP^2$ $(n = \sigma/a^2)$.

Thus, a solution of the stated problem should be sought in the form

$$\varphi(x, y, z, t) = T(x, y, z, t) + nP^2(x, y, z).$$

It is seen from Eq. (7) that the function T(x, y, z, t) is determined from the solution of two problems: the appropriate boundary problem of the heat conduction equation, and the problem of a stationary pressure distribution in the porous medium, the solution of which can be constructed on the basis of the results of [3].

As a result of this it is assumed that the temperature at each of the boundary sections remains constant, i.e., the surfaces corresponding to the entry and exit of gas are simultaneously isobaric and isothermal surfaces. At the moment t = 0 the temperature of the exit surface abruptly assumes a new constant value, as a result of which a redistribution of T and P takes place within the body.

NOTATION

T	is the temperature;
P	is the pressure;
ρ	is the density;
t	is the time;
v	is the velocity of filtration;
μ	is the coefficient of dynamic viscosity;
a^2	is the coefficient of thermal diffusivity;
k	is the coefficient of permeability;
R	is the gas constant;
c_p	is the heat capacity of a gas at constant pressure;
x, y, z	are spatial coordinates.

Subscript

1 denotes the porous medium.

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